Quantum Entanglement in Exactly Solvable Models(Topological Aspects of Solid State Physics)

Katsura, Hosho

DAY 4: 16:40 – 17:00

Critical properties at the Anderson transition in the two-dimensional Z_2 quantum spin-Hall system

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The quantum spin-Hall (QSH) effect occurs in a new kind of a topological insulator characterized by the Z_2 topological number. Since QSH system possesses time-reversal symmetry and broken spin-rotation symmetry, this system is expected to belong to the symplectic class if taking account of symmetries but ignoring the topological nature. Recently, it is discussed that the Anderson transition of the QSH system belongs to the different universality class by reflecting the topological nature.
In this work, we investigate various critical properties at the Anderson transition of the QSH system in two-dimensions (2D) by using the network model. By applying the finite size-scaling analysis for the localization length, it is found that the critical exponent ν characterizing the divergence of the localization length at the critical point is identified with that of the ordinary symplectic class in 2D.
We also investigate bulk and boundary multifractality in the QSH system, which characterizes scale-invariant nature of wave functions at criticality in the bulk and near the boundary, respectively. We found that bulk multifractality in the QSH system is same as that of the ordinary symplectic class in 2D. When open boundaries are imposed on the QSH system, there exist two kinds of critical points depending on whether a boundary induces edge states in the adjacent insulating phase. It is found that boundary multifractality at the critical point of the metal - ordinary insulator (absence of edge states) transition is also same as that of the ordinary symplectic class. On the other hand, boundary multifractality at the critical point of the metal-topological insulator (presence of edge states) transition is completely different from that of the ordinary symplectic class. Therefore, boundary multifractality observed at the latter critical point is considered to be a new boundary critical phenomenon in the symplectic class, reflecting the presence of topologically non-trivial edge states in the adjacent insulating phase.

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Quantum Entanglement in Exactly Solvable Models

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There is considerable current interest in studying interacting quantum systems from quantum information perspective. The relation between quantum entanglement and quantum phase transitions has been extensively studied. In these studies, exactly solvable models play an important role as a laboratory to test the newly introduced concepts such as entanglement entropy and topological order. In this talk, I will present our recent results for the two kinds of solvable models: i) generalized Affleck-Kennedy-Lieb-Tasaki(AKLT) model [1,2], and ii) Calogero-Sutherland model[3]. For the generalized AKLT model, we found that the reduced density matrix of a subsystem is exactly spanned by the edge states, i.e., the degenerate ground states of the open system. This is the reason why the entanglement entropy coincides with the logarithm of the number of the edge states. For the Calogero-Sutherland model, we studied the entanglement between two subsets of particles in the ground state. Using the duality relations of the Jack symmetric polynomials, we estimate an upper bound of the entanglement entropy and interpret it in terms of fractional exclusion statistics. This work was done in collaboration with Takaaki Hirano, Yasuyuki Hatsuda (U. Tokyo), Ying Hu, Vladimir E Korepin(SUNY,YITP) and Yasuhiro Hatsugai(U. Tsukuba).

[1] H. Katsura, T. Hirano and Y. Hatsugai, Phys. Rev. B 76, 012401(2007).
[2] Y. Xu, H. Katsura, T. Hirano and V. E. Korepin, arXiv:0802.3221[quant-ph], arXiv:0804.1741[quant-ph].
[3] H. Katsura and Y. Hatsuda, J. Phys. A: Math. Theor. 40, 13931(2007).